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GLANCING SHOCK WAVE AS A LIMITING CASE OF IRREGULAR REFLECTION

S. K. Andilevko UDC 534.2

The author proposes a computational method for a glancing shock wave (GSW) at the interface of two media with different acoustic impedances provided that the GSW travels in the medium with the lower impedance.

In [1, 2], the regularities of the reflection and refraction of skew shock waves (SSWs) at gas–gas and gas–solid interfaces were investigated; in [3], these results were generalized to surfaces of contact (SC) between various materials described by polytropes (with indices k for medium 1 and m for medium 2) or $D-U$ adiabats of the form $D = a + bU$ (with coefficients a, b and c, d for media 1 and 2, respectively). A common requirement was that the impedance and density of the half-space in which the SSW propagates (in what follows, simply medium 1) be lower than those in the medium, in which the reflected shock wave (RISW) (medium 2) propagates, i.e., $\rho_1 < \rho_4$, $\rho_1 c_1 < \rho_4 c_4$, $D > c_1$, and $D > c_4$. We make use of the computational procedure of [3] to investigate a limiting case of the interaction of an SSW with an SC, namely, a shock wave (SW) that slides along the SC of two media.

Using the Rankine–Hugoniot conditions and the equation of state, we will determine the parameters of medium 1 behind the GSW front in a coordinate system connected with the point of contact (PC) between the GSW and the SC (Fig. 1) for a polytropic gas (in what follows, PM) by the system of equations

$$p_2 = \frac{2\rho_1 D^2}{k+1} - \frac{k-1}{k+1} p_1 \, , \quad \frac{\rho_1}{\rho_2} = \frac{k+1+(k-1)\,p_2/p_1}{k-1+(k+1)\,p_2/p_1} = K\,(p_2/p_1) \, ,$$
$$q_2 = DK\,(p_2/p_1) \, , \quad \theta_2 = 0 \, , \tag{1a}$$

and for a substance described by the $D-U$ adiabat (in what follows, AM) by

$$p_2 = p_1 + \frac{\rho_1 D^2}{b}\left(1 - \frac{a}{D}\right) , \quad \frac{\rho_2}{\rho_1} = \frac{b}{b-1+a/D} , \quad q_2 = D\left(b - 1 + \frac{a}{D}\right) , \quad \theta_2 = 0 \, . \tag{1b}$$

It has been shown [3] that for PM–AM interfaces the angle of direct refraction φ_t is very close to $\pi/2$ but still is smaller than it, while for surfaces of contact of PM–PM and AM–AM types φ_t is substantially smaller than a right angle. According to [1-3], the form of irregular reflection that is characteristic of angles $\varphi \to \pi/2$ and the limiting case $\varphi = \pi/2$ will always correspond to the regime of strong shockless irregular reflection (SSIR) [1] when the convexity of the Mach wave (MW) (Fig. 1) faces the incoming flow and the center of curvature of MW is on the left of PC. For GSW ($\varphi = \pi/2$), the center of curvature lies on a line of tangential decay that is not rotated (more precisely, this is the surface that separates the region of p_m, ρ_m, and q_m that vary along the MW from the region of constant p_2, ρ_2, and q_2). The parameters of the flow in the vicinity of the SC in medium 1 are described by the system

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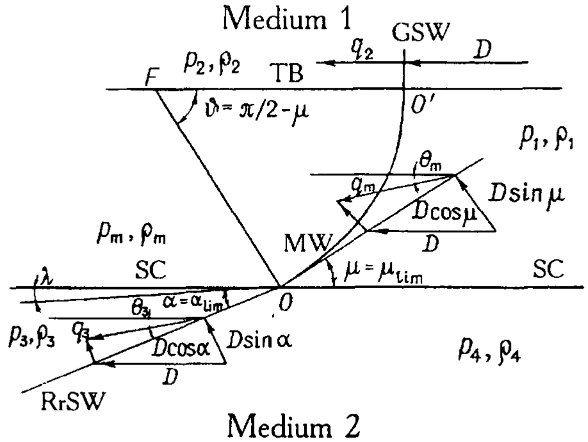

Fig. 1. Break decay in the interaction of a glancing shock wave with the surface of contact of two media.

$$p_m = \frac{2\rho_1 D^2}{k+1}\sin^2\mu - \frac{k-1}{k+1}p_1, \quad \frac{\rho_1}{\rho_m} = K(p_m/p_1), \tag{2a}$$

$$q_m = D\cos\mu\sqrt{1+\tan^2\mu K^2(p_m/p_1)}, \quad \theta_m = \mu - \arccos\sqrt{\left(\frac{1}{1+\tan^2\mu K^2(p_m/p_1)}\right)}$$

for a PM and by

$$p_m = p_1 + \frac{\rho_1 D^2}{b}\sin^2\mu\left(1 - \frac{a}{D\sin\mu}\right),$$

$$q_m = D\cos\mu\sqrt{\left(1 + \frac{\tan^2\mu}{b^2}\left(b - 1 + \frac{a}{D\sin\mu}\right)^2\right)}, \tag{2b}$$

$$\frac{\rho_m}{\rho_1} = \frac{b}{b-1+a/(D\sin\mu)}, \quad \theta_m = \mu - \arccos\sqrt{\left(\frac{1}{1+\frac{\tan^2\mu}{b^2}\left(b-1+\frac{a}{D\sin\mu}\right)^2}\right)}$$

for an AM. For an RrSW in medium 2,

$$p_3 = \frac{2\rho_4 D^2}{m+1}\sin^2\alpha - \frac{m-1}{m+1}p_1, \quad \frac{\rho_4}{\rho_3} = M(p_3/p_1) = \frac{m+1+(m-1)p_3/p_1}{m-1+(m+1)p_3/p_1}, \tag{3a}$$

$$q_3 = D\cos\alpha\sqrt{1+\tan^2\alpha M^2(p_3/p_1)}, \quad \theta_3 = \alpha - \arccos\sqrt{\left(\frac{1}{1+\tan^2\alpha M^2(p_3/p_1)}\right)}$$

for a PM and

$$p_3 = p_1 + \frac{\rho_4 D^2}{d}\sin^2\alpha\left(1 - \frac{c}{D\sin\alpha}\right),$$

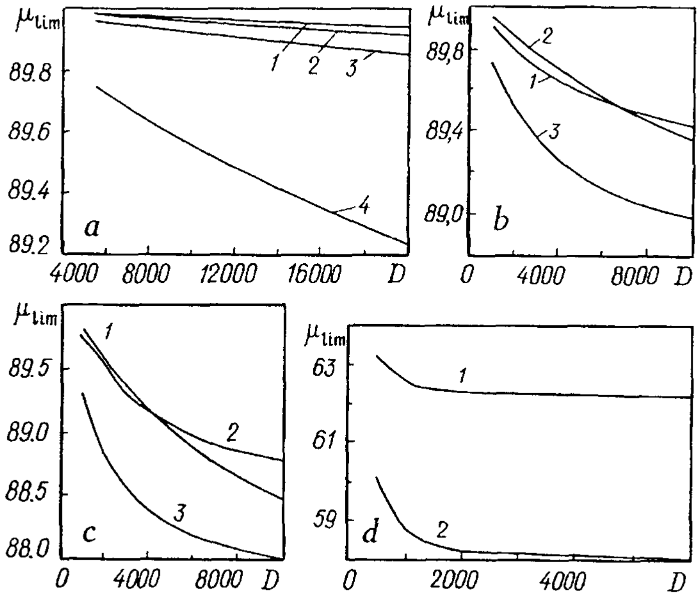

Fig. 2. $\mu_{\lim}$ (deg) vs. D (m/sec) for pairs of substances of the type: a) gas–continuum: 1) air–iron, 2) air–titanium, 3) krypton–iron, 4) air–water; b) air–W (1), Al_2O_3 (2), and Ni (3) powder; c) krypton–W (1), Al_2O_3 (2), and Ni (3) powder; d) gas–gas: 1) air–krypton, 2) air–propane.

$$q_3 = D\cos\alpha\sqrt{\left(1 + \frac{\tan^2\alpha}{d^2}\left(d - 1 + \frac{c}{D\sin\alpha}\right)^2\right)},$$

$$\frac{\rho_3}{\rho_4} = \frac{d}{d - 1 + c/[D\sin\alpha]},\quad \theta_3 = \alpha - \arccos\sqrt{\left(\frac{1}{1 + \frac{\tan^2\alpha}{d^2}\left(d - 1 + \frac{c}{D\sin\alpha}\right)^2}\right)},\tag{3b}$$

for an AM. The combined system of equations (1)-(3) contains 12 equalities for determination of 14 unknowns: $p_2, q_2, \rho_2, \theta_2, p_m, q_m, \rho_m, \theta_m, p_3, q_3, \rho_3, \theta_3, \mu$, and α. The equations that establish equality of the pressure and the normal flow velocities on the two sides of the SC

$$p_m = p_3,\quad q_m\sin\theta_m = q_3\sin\theta_3.\tag{4}$$

help to close system (1)-(3). The first equation of (4) establishes a relationship between α and μ:

$$\sin\alpha = \frac{c}{2D}\left[1 + \sqrt{\left(1 + 4\frac{d}{b}\frac{\rho_1}{\rho_4}\left(\frac{D}{c}\right)^2\sin^2\mu\left(1 - \frac{a}{D\sin\mu}\right)\right)}\right]\tag{5a}$$

for an AM–AM SC,

$$\sin\alpha = \sqrt{\left(2\frac{\rho_1}{\rho_4}\frac{m+1}{k+1}\sin^2\mu - \left(\frac{k-1}{k+1} - m + 1\right)\frac{p_1}{\rho_4 D^2}\right)}\tag{5b}$$

for PM–PM, and, finally,

$$\sin\alpha = \frac{c}{2D}\left[1 + \sqrt{\left(1 + \frac{2d}{k+1}\left(\frac{\rho_1}{\rho_4}\sin^2\mu - k\frac{p_1}{\rho_4 D^2}\right)\right)}\right]\tag{5в}$$

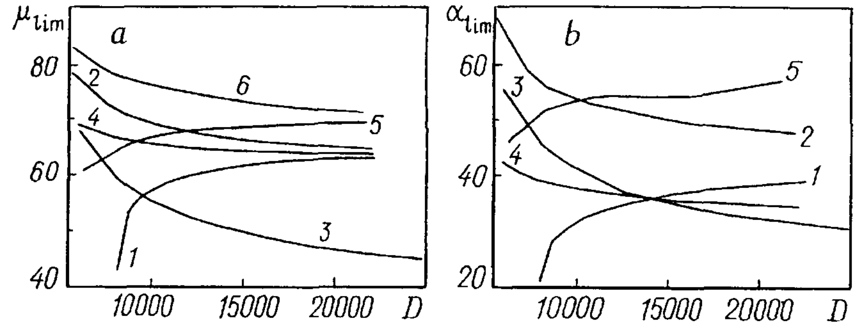

Fig. 3. μ_{lim} (a) and α_{lim} (b) (deg) vs. D (m/sec) for a surface of contact of AM–AM types: 1) titanium–lead; 2) titanium–cobalt, 3) aluminum–titanium, 4) aluminum–iron, 5) titanium–iron, 6) sand–iron.

for a PM–AM surface. The second equality of (4) becomes a transcendental equation for determination of the angle μ. It is purposeless to write here the explicit form of all three kinds of it (PM–PM, AM–AM, and PM–AM). Once μ is numerically determined, all remaining parameters of the flows can be calculated using relations (1-3) and (5), while the angle of SC rotation behind the PC is

$$\lambda = \arctan \frac{q_3 \sin \theta_3}{D} . \tag{6}$$

Thus, the break decay (BD) for an SW that slides along the SC will be calculated completely.

In the case where $D > c_1$ but $D < c_4$ the appearance of an RrSW in medium 2 is impossible, and the SC can be considered rigid (RS). System of equations (3) loses meaning together with conditions (4) and (5), while the requirement of parallelism of the flow and the RS ($\theta_m = 0$) brings system (2) to complete coincidence with (1). The angle $\mu = \pi/2$ [3]. System (1) describes completely the parameters of the flow behind the GSW, while the travel of the SC (in this case, an RS) behind the PC is calculated by evaluating the magnitude and duration of the pulse transfered by the RS due to the increase in the pressure behind the GSW front.

Figures 2 and 3a give the angles μ_{lim} that correspond to the angle of entry of the MW into the SC in BD that occurs as a result of GSW motion along the SC of various materials. In Fig. 2a these angles correspond to PM–AM-type SCs. The angle μ_{lim}, in this case, is very large and is only one-two tenths of a degree smaller than a right angle. Figure 2b and c gives the dependences $\mu_{lim}(D)$ for certain powders, whose adiabats are taken from [4] (data for all the remaining substances are taken from [5]), bordering air and krypton, respectively. We note that here, too, the values of μ_{lim} are only slightly less than 90^0. In Fig. 2d, the dependence $\mu_{lim}(D)$ is obtained for air in contact with krypton and propane (the PM–PM type of SC); the difference of μ_{lim} from a right angle is rather large. Figure 3a and b illustrates the dependences $\mu_{lim}(D)$ and $\lambda_{lim}(D)$, respectively, for various materials with a surface of contact of AM–AM type. Here, μ_{lim} differs from a right angle very significantly; it should be noted that μ_{lim} increases with ρ_4/ρ_1, and in the limit as $\rho_4/\rho_1 \to \infty$ the angle $\mu_{lim} \to \pi/2$.

All the above indicates that a glancing shock wave on the surface of contact of two media that travels over the medium with a lower impedance can be considered to be a limiting case of irregular reflection (SSIR) according to the classification of [1-3]. The break decay can be calculated accurately within the framework of applicability of the hydrodynamic theory of shock waves.

NOTATION

D, shock-wave velocity; c, velocity of sound in the medium (always with a subscript); p, pressure; q, total flow velocity in a coordinate system connected with the point of intersection of the skew shock wave and the surface; k and m, polytrope indices of the gas for medium 1 and medium 2, respectively; a, b and c, d, coefficients of the $D-U$ adiabats for medium 1 and medium 2, respectively (c here is always without a subscript); ρ, density; α, slope of the refracted shock wave to the SC; φ, slope of the skew wave to the SC; λ, angle of rotation of the SC behind the PC; μ, angle of entry of the Mach wave into the point of contact of the skew shock wave with the SC; θ, angle of rotation of the vector of the total velocity of the flow behind the shock-wave front; θ, angle of opening of the sector of a circumference that approximates the Mach-wave surface. Subscripts: 1, initial parameters of medium 1; 2, parameters of medium 1 (behind the glancing shock wave); 4, initial parameters of medium 2; t, transition angle between regimes of irregular and regular interaction (the angle of direct refraction); m, parameters of the flow in the lower portion of the Mach wave at the angle μ; lim, at μ and α for the limiting case $\varphi = \pi/2$.

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